which leads to q = 0 at the bounding planes and

$$q = (E_{01} - E_{02})\,(3k\,\delta/4)^{-1}$$

inside the layer.

This contradictory result is due to the incorrectly posed boundary conditions, as the authors allow radiative nonequilibrium at the bounding planes of an equilibrium layer. This is inconsistent with (2) and leads to the above contradiction.

The contradiction can be eliminated by deriving the equation for q from the assumption of radiative equilibrium of a boundary layer of thickness $1/k$ [6]. The formulation of the radiative equilibrium of the boundary layer requires the application of Buguer's law. Thus, Adrianov and Polyak's remark that the boundary conditions derived by the author contradict Buguer's law is incorrect.

The incorrectness of the boundary condition in [5] is also indicated by the fact that the solution of the radiative heat transfer problem in cylindrical and spherical gas layers does not, under these conditions, lead in the limit to Christiansen's formula.

Further, Adrianov and Polyak define the diffusion coefficient of radiative energy transfer by the expression $D = c/3k$, which is incorrect. Simple calculation shows that for a plane layer of a gray gas $D = c/4k$. The derivation of this formula can be found in several papers [6, 7].

Now as regards the comparison between my solution and the "exact" solution of Hottel [2].

A study of Hottel's graph [2] shows that for values $k\delta \geq 6$ my solution practically coincides with Hottel's "exact" solution. Thus Adrianov and Polyak's remark that for large $k\delta$ my solution leads to results 25% lower than Hottel's "exact" values is incorrect.

In conclusion, I would like to remark that there is no sufficient foundation for regarding Hottel's solution as absolutely exact, as the mathematical formulation of the problem considered by Hottel is by no means clear. It can be said only that correct application of the differential and integral approaches to radiative heat transfer should lead to identical results.

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HEAT TRANSFER ON A NONISOTHERMAL FLAT PLATE WITH A LAMINAR BOUNDARY LAYER

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Reference [1] contains the very interesting results of an experimental investigation of heat transfer in longitudinal flow over a nonisothermal flat plate. In particular, data are given for the case of an initial adiabatic section followed by a region with approximately constant heat flux. Under these conditions (which had not been previously examined experimentally) the unheated length was observed to have an appreciable effect on heat transfer in the presence of a laminar boundary layer.

This effect is described in [1] by the empirical relation:

$$K_{0,x_1} = 0.43\,[x_1/x]^{1/6},$$
$$K_{0,x_1} = \mathrm{Nu}_{f,x_1}\,\mathrm{Re}_{f,x_1}^{-1/2}\,\mathrm{Pr}_f^{-1/3}\,[\mathrm{Pr}_f/\mathrm{Pr}_w]^{-1/4}. \tag{1}$$

The possibility of correlating the experimental data with theory is of considerable interest. In the report of the Kaunas Symposium on Convective Heat Transfer in Single-Phase Media (1962), the present author showed that the local heat flux density on a nonisothermal plate can be represented by

$$q_w(x) = K_{0,x}\,\lambda\,T_w(x)\,[w_\infty/\nu\,x]^{1/2}\,\mathrm{Pr}^{1/3}. \tag{2}$$

Here the coefficient $K_{0,x}$ depends on the temperature conditions (i. e., the form of function $T_w(x)$:

$$K_{0,x} = 0.33\left[1 + \frac{\int\limits_0^z T_w dz}{2zT_w}\,\frac{d(\ln T_w)}{d(\ln z)}\right]\left[\frac{zT_w}{\int\limits_0^z T_w dz}\right]^{1/3} \tag{3}$$

(here $z \equiv x^{3/4}$).

This solution was obtained by means of an iterative sequence of integration of the differential equations of the laminar boundary layer and is, generally speaking, approximate. The specific advantage of this method over other known methods of calculating isothermal heat transfer [2-7] is the possibility of calculating $K_{0,x}$ in explicit form for a wide range of functions describing the surface temperature. It has been shown that calculations based on Eq. (3) give satisfactory results for all forms of temperature nonuniformity examined experimentally and analytically.

A theoretical estimate of $K_{0,x}$ for the experimental conditions of [1] may be obtained from (3) if we assume

$$\text{when } x < x_0 \quad T_w(x) \equiv 0,$$
$$\text{when } x \geqslant x_0 \quad T_w(x) \sim x^n. \tag{4}$$

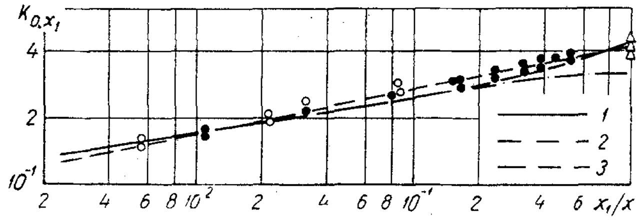
Integrating in (3) and then going over in (2) from the coordinate x to x_1, as in [1], we have

$$K_{0,x_1} = 0{,}33\,\frac{(1+2n)\,[x_1/x]^{1/2}}{(1+4n/3)^{2/3}}\left\{1 - \frac{2n/3}{1+2n}\left[1-\frac{x_1}{x}\right]^{n+3/4}\right\}\times$$
$$\times\left\{1-\left[1-\frac{x_1}{x}\right]^{n+3/4}\right\}^{-1/3}, \tag{5}$$
$$(K_{0,x_1}\,[x_1/x]^{1/2} \equiv (K_{0,x}).$$

It can be shown that to make an approximate allowance for the conditions q_w = const on the section x_1 for any values of x_1/x it is sufficient to substitute a constant value of the exponent n = 1/2 in (5) (or the value n = 0.4 from the experiments of [1] for x_1 = x). Actually, when x_1/x is close to unity, such a value of n satisfies the condition q_w = = const quite well (this is obvious). For small values of x_1/x other values of n must generally be taken to satisfy this condition. However, for small x_1/x, (5) is invariant with respect to n if

$$\text{when } x_1 \ll x \quad K_{0,x_1} = 0.33\,(4/3)^{1/3}\,[x_1/x]^{1/6}.$$

The figure gives the results of calculations based on (5) for n = 0.4 in the coordinates $K_{0,x_1} = f(x_1/x)$. The experimental points of [1] are also given, together with the line corresponding to the empirical formula (1).



$K_{0,x_1} = f(x_1/x)$: 1, 2) calculated from (5) for n = 0.4 and n = 0, 3) from (1). Experimental points from [1].

The data presented show that the theoretical calculation is in good agreement with experiment over the entire range of x_1/x examined.

A table is given in [1] in which the relative variation of K_{0,x_1} as a function of x_1/x is given both for empirical relation (1) and also for Eckert's theoretical relation [6]:

$$K_{0,x_1} = 0.33[x_1/x]^{1/2}\{1-[1-x_1/x]^{3/4}\}^{-1/3}. \quad (7)$$

The variation of these coefficients was found to differ. It is clear from the above analysis that the disparity is due to the fact that the comparative variations relate to different kinds of temperature nonuniformity. It is, therefore, incorrect to state that Eckert's relation is inexact or requires correction. His equation, as is known [6, 7], was obtained for a stepwise temperature nonuniformity, when T_W = const on the heat transfer section. This relation (7) may be obtained in particular, from solution (5) by putting n = 0 (see figure).

It may be concluded from the above remarks that the method recommended in [1] for calculating heat transfer with a stepwise temperature nonuniformity is only valid for the case examined when $q_W \simeq$ const on the heat transfer section. When T_W = const on the heat transfer section (Eckert nonuniformity), calculations based on (1) will give a reduced heat transfer rate. Evidently, for both types of step nonuniformity (and also for intermediate conditions) calculation of K_{0,x_1} according to (5), with suitable values of the exponent n, will give satisfactory results.

NOTATION

x_1 and x – heated and total length of plate; K_{0,x_1} – relative local heat transfer rate (taking into account correction for variation of thermophysical properties); $T_W(x)$ – local temperature head; x_0 – length of initial unheated section.

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